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DIFFUSION-THERMAL INSTABILITY OF A LAMINAR FLAME

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The problem of the diffusion-thermal instability of a laminar flame with a nonzero thickness of the chemical-reaction zone is solved. Two scenarios of the behavior of the process of burning in the presence of small perturbations with constant and variable velocities of motion of the flame are considered. The latter, in particular, leads to a Markstein formula relating the velocity of motion of the burning front to the curvature of this front and virtually to the absolute instability of burning. In the case of a constant velocity of the flame and the Lewis number Le > 1, only the aperiodic loss of stability is implemented, whereas in the case of Le < 1 only the periodic loss is implemented.

In the most general formulation of the problem of investigating diffusion-thermal instability, the behavior of burning is considered with respect to arbitrary perturbations resulting in the spatial distortion of a plane laminar flame that moves relative to the initial combustible mixture with velocity v_{n}, which is called the normal velocity of propagation. The instability of the laminar flame has been investigated in a great number of works ([1–5] and more]. The first mathematical investigation of the so-called one-dimensional stability was carried out by Rosen [1], whose conclusions were disproved by Barenblatt, Zel'dovich, and Kanel [2, 3]. In [6], it is assumed that the flame is unstable when distorted and when the effective Lewis numbers are $\mathrm{Le} < 1$ ($\mathrm{Le} = D/\kappa$); the instability is caused by the enthalpy excess in the burning front. But if $\mathrm{Le} > 1$, the burning is stable. The mathematical calculation presented in [4] (in this work, the zone of chemical reaction was considered to be infinitely thin) led to the opposite conclusion: the burning is stable when $\mathrm{Le} < 1$ and unstable when $\mathrm{Le} > 1$. More recent [5, 7, and others] investigations, similar to those conducted in [4], have shown that the diffusion-thermal instability of the laminar flame can be observed for Lewis numbers both more and less than unity. It turns out that the region of stable burning is determined not only by the value of the Lewis number but also by the parameter $\psi = E(T_{\mathrm{b}} - T_0)/(2RT_{\mathrm{b}}^2) >> 1$, which was present even in [4] and characterizes the chemical-reaction rate. For rather large values of this parameter the region of stable burning is localized in a small neighborhood near the straight line $\mathrm{Le} = 1$ on the plane (Le, ψ), which is the unique stability condition within the limit $\psi \to \infty$ [7, 8].

Plane Flame Front. In the simple case of the gross reaction of first order, the rate of the chemical reaction W is described by the formula

$$W = Nk_0 \exp\left(-\frac{E}{RT}\right), \tag{1}$$

which is a particular form of the Arrhenius law. An analytical investigation of the problems of nonstationary burning with application of Eq. (1) is very laborious. Therefore, instead of Eq. (1) we take the more simplified form

$$W = Nk_0 \exp\left(-\frac{E}{RT_{\mathrm{b}}}\right)\eta\,(T - T_*) \tag{2}$$

with the ignition temperature T_*, whose form is determined from the requirement on agreement between the plane-flame velocity and the similar expression from the Zel'dovich–Frank-Kamenetskii theory at high activation energies. The reaction rate (2) retains the most important properties of the Arrhenius law, i.e., its strong temperature dependence and nonlinearity. Formula (2) was successfully used [9] in analytical calculations of the velocity of motion of the plane stationary flame front at arbitrary Lewis numbers.

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With the use of Eq. (2), the plane stationary flame front in a gaseous mixture with the chemical gross reaction of first order occurring in this mixture is described by the system of equations

$$v_n \frac{dT}{dx'} = \kappa \frac{d^2T}{dx'^2} + \frac{Q}{c_p} N k_0 \exp\left(-\frac{E}{RT_b}\right) \eta\,(T - T_*)\,, \quad v_n \frac{dN}{dx'} = D \frac{d^2N}{dx'^2} - N k_0 \exp\left(-\frac{E}{RT_b}\right) \eta\,(T - T_*)\,. \tag{3}$$

With the introduction of the dimensionless parameters and the velocity scale v_*

$$u = \frac{T - T_0}{T_b - T_0} = \frac{c_p}{QN_0}(T - T_0)\,, \quad b = \frac{N_0 - N}{N_0}\,, \quad x = \frac{x' v_*}{\kappa}\,, \quad w = \frac{v_n}{v_*}\,, \quad T_b = T_0 + \frac{Q}{c_p} N_0$$

system (3) takes the form

$$w^0 \frac{du}{dx} = \frac{d^2u}{dx^2} + W\,, \quad w^0 \frac{db}{dx} = \mathrm{Le} \frac{d^2b}{dx^2} + W\,, \quad W = a\,(1 - b)\,\eta\,(u - u_*)\,, \quad a = \frac{k_0\kappa}{v_*^2} \exp\left(-\frac{E}{RT_b}\right), \tag{4}$$

in which w^0 is the dimensionless velocity of the plane stationary flame. In what follows, the superscript 0 will mean that the symbol belongs to this type of flame.

According to (4), the problem of propagation of the flame corresponds to the following boundary conditions:

$$x \to -\infty: \quad u^0 = b^0 = 0\,; \quad x \to +\infty: \quad du^0/dx = db^0/dx = 0\,. \tag{5}$$

Locating the site of rupture [Eq. (2)] at the point $x = 0$ and assigning indices 1 and 2 to the temperature u^0 and the burn-out b^0 for $x < 0$ and $x > 0$, respectively, we give the distributions of $u^0(x)$ and $b^0(x)$:

$$x < 0: \quad u_1^0 = u_* \exp(w^0 x) = \frac{k}{k + w^0} \exp(w^0 x)\,, \quad b_1^0 = \left(1 - \frac{w^0 k}{a}\right) \exp\left(\frac{w^0 x}{\mathrm{Le}}\right) = \frac{k^2\,\mathrm{Le}}{a} \exp\left(\frac{w^0 x}{\mathrm{Le}}\right),$$

$$x > 0: \quad u_2^0 = 1 - \frac{w^0}{k + w^0} \exp(-kx)\,, \quad b_2^0 = 1 - \frac{w^0 k}{a} \exp(-kx)\,, \quad k = \frac{\sqrt{(w^0)^2 + 4a\,\mathrm{Le}} - w^0}{2\,\mathrm{Le}}\,, \tag{6}$$

$$a = \frac{k_0\kappa}{v_*^2} \exp\left(-\frac{E}{RT_b}\right), \quad (w^0)^2 = \left(\frac{1 - u_*}{u_*}\right)^2 \frac{a}{\mathrm{Le} + (1 - u_*)/u_*}\,,$$

where k is the positive root of the equation $\mathrm{Le}\;k^2 + w^0 k - a = 0$.

Solutions (6) satisfy boundary conditions (5) and the conditions of continuity of $u^0(x)$ and $b^0(x)$ and of their first derivatives at the point $x = 0$.

In the limit $a \to \infty$; this corresponds to an infinitely high activation energy. Then $u_* \to 1$, and assuming that

$$\frac{1 - u_*}{u_*} \approx 1 - u_* = \frac{T_b}{T_b - T_0} \sqrt{\frac{2T_0}{T_b} \frac{RT_b}{E}}\,,$$

we obtain the expression for the flame velocity v_n given in [8]. It is convenient to take v_n as the velocity scale v_*. Then

$$w^0 = 1\,, \quad a = \mathrm{Le}\,n \left(\frac{n - 1}{n} \frac{E}{RT_b}\right)^2, \quad n = \frac{T_b}{T_0}\,.$$

With consideration of the aforesaid, hereafter we will assume that $w^0 = 1$.

With the taken form of the ignition temperature u_* in the limit $E/RT_b \to \infty$ the rate of the chemical reaction in the form of Eq. (2) tends to the Dirac δ function and approximates arbitrarily exactly the similar expression by the Arrhenius law.

Mathematical Formulation of the Problem for a Slightly Distorted Flame. Now we superimpose a small perturbation on the plane stationary flame; this perturbation causes the deformation (identical in magnitude) of its front along the transverse coordinates y, z. To perform an analysis linear in perturbation degrees, it is sufficient to consider the system of equations

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial x} = \Delta u + W\,, \quad \frac{\partial b}{\partial t} + w\frac{\partial b}{\partial x} = \mathrm{Le}\,\Delta b + W\,, \quad \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\,, \tag{7}$$

in which the convective transfers in the directions y and z are neglected as small quantities of higher orders. The dimensionless time t is measured in the units κ/u_n^2. We will seek the solutions of Eq. (7) in the form

$$u = u^0(x) + \xi(y, z, t)\,F(x)\,, \quad b = b^0(x) + \xi(y, z, t)\,G(x)\,, \quad w = w^0 + w' = w^0 - q\Delta'\xi\,, \quad w' = -q\Delta'\xi\,,$$

$$\Delta' = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\,,$$

where the deformation $\xi(y, z, t)$ of the burning front has the exponential time dependence with the perturbation-growth increment Ω (or the perturbation-growth index) and the sine dependence on the spatial coordinates with the wave numbers λ_1 and λ_2 in the directions y and z: $\partial\xi/\partial t = \Omega\xi$ and $\Delta'\xi = -\lambda^2\xi$; $\lambda^2 = \lambda_1^2 + \lambda_2^2$.

The physical meaning of the parameter q, which, in the stationary regime of burning, is the Markstein constant, will be defined below.

The substitution of u, b, and w into Eq. (7), further linearization, and application of Eq. (2) in intermediate calculations after simple transformations give the equations for F and G:

$$\frac{d^2F}{dx^2} - \frac{dF}{dx} - (\Omega + \lambda^2)\,F + \frac{\partial W}{\partial u^0}F = -\frac{\partial W}{\partial b^0}G + q\lambda^2\frac{du^0}{dx}\,,$$
$$\mathrm{Le}\,\frac{d^2G}{dx^2} - \frac{dG}{dx} - (\Omega + \lambda^2\,\mathrm{Le})\,G + \frac{\partial W}{\partial b^0}G = -\frac{\partial W}{\partial u^0}F + q\lambda^2\frac{db^0}{dx}\,. \tag{8}$$

Having calculated the derivatives entering into Eq. (8)

$$\frac{\partial W}{\partial u^0} = a\,(1 - b^0)\,\delta\,(u^0 - u_*)\,, \quad \frac{\partial W}{\partial b^0} = -a\eta\,(u^0 - u_*)\,,$$

next we have

$$\frac{d^2F}{dx^2} - \frac{dF}{dx} - (\Omega + \lambda^2)\,F + a\,(1 - b^0)\,\delta\,(u^0 - u_*)\,F = a\eta\,(u^0 - u_*)\,G + q\lambda^2\frac{du^0}{dx}\,,$$
$$\mathrm{Le}\,\frac{d^2G}{dx^2} - \frac{dG}{dx} - (\Omega + \lambda^2\,\mathrm{Le})\,G - a\eta\,(u^0 - u_*)\,G = -a\,(1 - b^0)\,\delta\,(u^0 - u_*)\,F + q\lambda^2\frac{db^0}{dx}\,. \tag{9}$$

Boundary conditions (5) also hold for the perturbed flame, only in Eq. (5) we must replace $u^0 \to u$ and $b^0 \to b$. The conditions obtained in this way for system (9) yield the vanishing of F and G for $x \to -\infty$ and of their first derivatives for $x \to +\infty$. At the point $x = \xi$ and within the framework of the linear analysis used here, the quantities u, b, du/dx, and db/dx must be continuous. The condition of continuity of u and b gives the equalities

$$F_1 = F_2 \, , \quad G_1 = G_2 \, . \tag{10}$$

The condition of continuity of their derivatives (more precisely, of the fluxes of energy and a substance) leads to the requirements

$$\frac{d^2 u_1^0}{dx^2} + \frac{dF_1}{dx} = \frac{d^2 u_2^0}{dx^2} + \frac{dF_2}{dx} \, , \quad \frac{d^2 b_1^0}{dx^2} + \frac{dG_1}{dx} = \frac{d^2 b_2^0}{dx^2} + \frac{dG_2}{dx} \, . \tag{11}$$

The second derivatives of $u^0(x)$ and $b^0(x)$ have a discontinuity at the point $x = 0$

$$\frac{d^2 u_1^0}{dx^2} - \frac{d^2 u_2^0}{dx^2} = k \, , \quad \frac{d^2 b_1^0}{dx^2} - \frac{d^2 b_2^0}{dx^2} = \frac{k}{\mathrm{Le}} \, ,$$

and then conditions (11) in final form appear as

$$\frac{dF_1}{dx} - \frac{dF_2}{dx} + k = 0 \, , \quad \frac{dG_1}{dx} - \frac{dG_2}{dx} + \frac{k}{\mathrm{Le}} = 0 \, . \tag{12}$$

Furthermore, the first derivatives of $F(x)$ and $G(x)$ undergo, in addition to Eq. (12), a discontinuity at the point $x = 0$, which immediately follows from Eq. (9). Integrating Eq. (9) over the vanishingly small region near $x = 0$ and using the known properties of the δ function [10], we find

$$\frac{dF_2}{dx} - \frac{dF_1}{dx} + (k+1)\, F_2 = 0 \, , \quad \frac{dG_2}{dx} - \frac{dG_1}{dx} + \frac{k+1}{\mathrm{Le}}\, F_2 = 0 \, . \tag{13}$$

Taking Eq. (10) into account, it is easy to see that Eqs. (12) and (13) simultaneously lead to the equalities

$$F_2 = -\frac{k}{k+1} = F_1 \, . \tag{14}$$

This means that one of equalities (14) can be taken as a boundary condition supplementary to Eqs. (10) and (12). Thus, when $x = 0$, we have five boundary conditions for determining the unknown four constants of integration (after satisfaction of the conditions for $x \to \pm\infty$) and the eigenvalues q and Ω.

Characteristic Equation. We now proceed to solution of Eqs. (9). For the region $x < 0$ from Eq. (9) with account for Eq. (6) we obtain the system of equations

$$\frac{d^2 F_1}{dx^2} - \frac{dF_1}{dx} - (\Omega + \lambda^2)\, F_1 = q\lambda^2 \frac{k}{k+1} \exp(x) \, , \quad \mathrm{Le} \frac{d^2 G_1}{dx^2} - \frac{dG_1}{dx} - (\Omega + \lambda^2\, \mathrm{Le})\, G_1 = q\lambda^2 \frac{k^2}{a} \exp\left(\frac{x}{\mathrm{Le}}\right).$$

The solutions of this system vanishing for $x \to -\infty$ are as follows:

$$F_1 = f_1 \exp(\alpha x) - q \frac{\lambda^2}{\Omega + \lambda^2} \frac{k}{k+1} \exp(x) \, , \quad \alpha = \frac{1 + \sqrt{1 + 4(\Omega + \lambda^2)}}{2} \, ,$$

$$G_1 = g_1 \exp(\beta x) - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} \frac{k^2}{a} \exp\left(\frac{x}{\mathrm{Le}}\right), \quad \beta = \frac{1 + \sqrt{1 + 4\, \mathrm{Le}\, (\Omega + \mathrm{Le}\, \lambda^2)}}{2\, \mathrm{Le}} \, . \tag{15}$$

Similarly, in the region $x > 0$ we have, respectively,

$$\frac{d^2F_2}{dx^2} - \frac{dF_2}{dx} - (\Omega + \lambda^2)\, F_2 = aG_2 + q\lambda^2 \frac{k}{k+1} \exp(-kx) \,,$$

$$\mathrm{Le}\, \frac{d^2G_2}{dx^2} - \frac{dG_2}{dx} - (\Omega + \mathrm{Le}\, \lambda^2)\, G_2 - aG_2 = q\lambda^2 \frac{k^2}{a} \exp(-kx) \,,$$

$$G_2 = g_2 \exp(-\chi x) - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} \frac{k^2}{a} \exp(-kx) \,, \quad \chi = \frac{-1 + \sqrt{1 + 4\, \mathrm{Le}\, (a + \Omega + \lambda^2\, \mathrm{Le})}}{2\, \mathrm{Le}} \,, \tag{16}$$

$$F_2 = f_2 \exp(-\gamma x) + A_1 g_2 \exp(-\chi x) - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} A_2 \exp(-kx) \,,$$

$$\gamma = \frac{-1 + \sqrt{1 + 4\, (\Omega + \lambda^2)}}{2} \,, \quad A_1 = \frac{a}{\chi^2 + \chi - (\Omega + \lambda^2)} \,, \quad A_2 = \frac{k}{k+1} \frac{k\,(k+1) - (\Omega + \mathrm{Le}\, \lambda^2)}{k\,(k+1) - (\Omega + \lambda^2)} \,.$$

In Eqs. (15) and (16), f_1, f_2, g_1, and g_2 are the constants of integration.

The substitution of Eqs. (15) and (16) into boundary conditions (10) results in the algebraic equations

$$f_1 - q \frac{\lambda^2}{\Omega + \lambda^2} \frac{k}{k+1} = f_2 + g_2 A_1 - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} A_2 \,, \quad g_1 = g_2 \,. \tag{17}$$

The use of Eqs. (15) and (16) in Eq. (12) yields the expressions

$$\alpha f_1 - q \frac{\lambda^2}{\Omega + \lambda^2} \frac{k}{k+1} + \gamma f_2 + \chi g_1 A_1 - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} kA_2 + k = 0 \,, \tag{18}$$

$$\beta g_1 - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} \frac{k^2}{a\, \mathrm{Le}} + \chi g_2 - q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} \frac{k^3}{a} + \frac{k}{\mathrm{Le}} = (\beta + \chi)\, g_1 - \left(q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} - 1 \right) \frac{k}{\mathrm{Le}} = 0 \,,$$

here, in writing the second expression, we applied the second equality from Eq. (17) and the above-given equality (see Eq. (6)) $\mathrm{Le}\ k^2 + k - a = 0$.

From the second expression in Eq. (18) we find

$$g_1 = \frac{k}{\mathrm{Le}\, (\chi + \beta)} \left(q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} - 1 \right). \tag{19}$$

The quantities f_1 and f_2 are determined from conditions (14):

$$f_1 = \frac{k}{k+1} \left(q \frac{\lambda^2}{\Omega + \lambda^2} - 1 \right), \quad f_2 = q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} - A_1 g_1 - \frac{k}{k+1} \,. \tag{20}$$

The first expressions in Eqs. (17), (18) and (19), (20) after simple calculation give the equation

$$\frac{k}{k+1} \left[q\gamma \frac{\lambda^2}{\Omega + \lambda^2} - \alpha - \gamma \right] + \varphi_1 \left(q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} - 1 \right) - \varphi_2 q \frac{\lambda^2}{\Omega + \mathrm{Le}\, \lambda^2} + k = 0 \,,$$

(21)

$$\varphi_1 = \frac{k}{\mathrm{Le}} \frac{\chi - \gamma}{\chi + \beta} \frac{k\,(\mathrm{Le}\,k + 1)}{\chi^2 + \chi - (\Omega + \lambda^2)}\,, \quad \varphi_2 = \frac{k\,(k - \gamma)}{k + 1} \frac{k\,(k + 1) - (\Omega + \mathrm{Le}\,\lambda^2)}{k\,(k + 1) - (\Omega + \lambda^2)}\,.$$

From the condition of satisfaction of the required equality in Eq. (21) it is necessary to determine the explicit form of q and Ω. The parameter q determines the change in the velocity of propagation of the flame in the presence of a perturbation in the initially plane front. To find a new value of the flame velocity, we must solve Eq. (21) for different values of q and Ω, assuming q to be the same root of Eq. (21) as Ω. Here, as the solution of problem (1) we obtain a set of pairs q and Ω. Using a specific example, we explain the manner in which the procedure of finding these eigenvalues is carried out. Let, for simplicity, $\mathrm{Le} = 1$. Then

$$\varphi_1 = \frac{\chi - \gamma}{\chi + \beta} \frac{k^2\,(k + 1)}{\chi^2 + \chi - (\Omega + \lambda^2)} = k\,\frac{\chi - \gamma}{\chi + \beta}\,, \quad \varphi_2 = \frac{k\,(k - \gamma)}{k + 1}\,,$$

and instead of Eq. (21) we have

$$k\left(q\,\frac{\lambda^2}{\Omega + \lambda^2} - 1\right)\left(\frac{\chi - \gamma}{\chi + \beta} - \frac{k - 2\gamma}{k + 1}\right) = 0\,. \tag{22}$$

The equality in Eq. (22) is satisfied when

$$q = \frac{\Omega}{\lambda^2} + 1 \tag{23}$$

and for such values of Ω which are the roots of the equation

$$\frac{\chi - \gamma}{\chi + \beta} - \frac{k - 2\gamma}{k + 1} = 0\,.$$

Solving this equation, we find that $\Omega_1 = -\lambda^2$ and $\Omega_2 = -\lambda^2 - 1/4$. The substitution of these quantities into Eq. (23) gives $q(\Omega_1) = 0$ and $q(\Omega_2) = -1/(4\lambda^2)$.

With account for formula (23) the determination of the flame velocity $w = w^0 - q\Delta'\xi$ now takes the form

$$w = w^0 - \left(\frac{\Omega}{\lambda^2} + 1\right)\Delta'\xi\,.$$

We replace here Ω and λ^2 by their expressions in terms of the derivatives of the front deformation in conformity with the equalities $\Omega = \frac{1}{\xi}\frac{\partial \xi}{\partial t}$ and $\lambda^2 = -\frac{1}{\xi}\Delta'\xi$:

$$w = w^0 + \frac{\partial \xi}{\partial t} - \Delta'\xi\,. \tag{24}$$

In this case, the first term $w^0 = 1$ is the velocity of the plane unperturbed flame, the second term is the kinematic additive, whose appearance is attributed to the fact that the problem is solved relative to the coordinate system tied to the plane unperturbed flame, and the third term determines the change in the flame velocity as a function of the curvature of its front (Markstein's correction [11, 12]).

The problem of the diffusion-thermal stability of the laminar flame can be formulated in two different ways. In the first of them, the velocity of propagation v_n of the flame can be considered to be constant in the presence of perturbations. Then for $q = 0$ the stability is determined by the absence of the increment Ω with a positive real part in the roots of (21). This approach has been applied in [4, 5, 7]. But the second approach used above is based on the

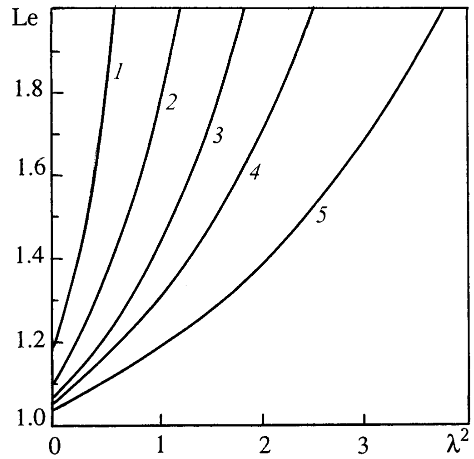

Fig. 1. Change in the region of aperiodic stability loss (Le = 1) with increase in the dimensionless activation energy k: 5 (1), 10 (2), 15 (3), 20 (4), and 30 (5).

assumption of variability of the flame velocity ($q \neq 0$) in the perturbed flame. Which of these approaches describes adequately the actual flame remains to be identified. However, it is clear that in the first case a limitation on the range of possible values of Ω is imposed.

Stability of Burning at a Constant Flame Velocity. Analysis of the stability of the flame at $v_n = \text{const}$ ($q = 0$) has been studied in sufficient detail [8]. Therefore, this problem is considered here only superficially. Assuming the Lewis number to be arbitrary, we consider Eq. (21) in the limit $k \to \infty$. The equation

$$q = \left(\frac{\Omega}{\lambda^2} + \mathrm{Le}\right) \frac{1/\mathrm{Le} - \beta + \gamma}{1/\mathrm{Le} - \beta + \gamma + \lambda^2 (\mathrm{Le} - 1)/\alpha}$$

obtained in this way for $q = 0$ falls into the following two equations: $\Omega/\lambda^2 + \mathrm{Le} = 0$ and $1/\mathrm{Le} - \beta + \gamma = 0$, whence we find that $\Omega_1 = -\mathrm{Le}\,\lambda^2$ and $\Omega_2 = \lambda$. The second, positive, root indicates the absolute instability of the flame in the case of infinitely high activation energy. This result has been obtained earlier in [7], but it also follows from the dispersion relation of [4].

Now we find the expression for the frequency of the unstable solution accurate to small quantities of the order of $1/k$ inclusive. When $q = 0$, Eq. (21) gives

$$\frac{k - 2\gamma}{k + 1} - \frac{\chi - \gamma}{\chi + \beta} \frac{k\,(\mathrm{Le}\,k + 1)}{\mathrm{Le}\,[\chi^2 + \chi - (\Omega + \lambda^2)]} = 0\,. \tag{25}$$

It should be noted that this characteristic equation for $k >> 1$ does not become one of the similar expressions obtained in [4, 7].

We expand Eq. (25) into a Taylor series:

$$\chi \approx k\left(1 + \frac{\Omega + \mathrm{Le}\,\lambda^2}{2\,\mathrm{Le}\,k^2}\right), \quad \chi^2 + \chi \approx k\,(k + 1)\left[1 + \frac{\Omega + \mathrm{Le}\,\lambda^2}{\mathrm{Le}\,k\,(k + 1)}\right],$$
$$\frac{k\,(\mathrm{Le}\,k + 1)}{\chi^2 + \chi - (\Omega + \lambda^2)} \approx \frac{\mathrm{Le}\,k + 1}{k + 1}\left(1 + \frac{\mathrm{Le} - 1}{\mathrm{Le}} \frac{\Omega}{k^2}\right), \quad \frac{\chi - \gamma}{\chi + \beta} \approx \left[1 - \frac{\gamma + \beta}{\chi} + \frac{\beta\,(\gamma + \beta)}{\chi^2}\right]. \tag{26}$$

After simple calculations with the use of Eq. (26), Eq. (25) is reduced to the form

$$\frac{1}{\mathrm{Le}} - \beta + \gamma + \frac{1}{k}\left[(\beta + \gamma)\left(\beta - \frac{1}{\mathrm{Le}}\right) + \frac{\mathrm{Le} - 1}{\mathrm{Le}}\,\Omega\right] = 0\,. \tag{27}$$

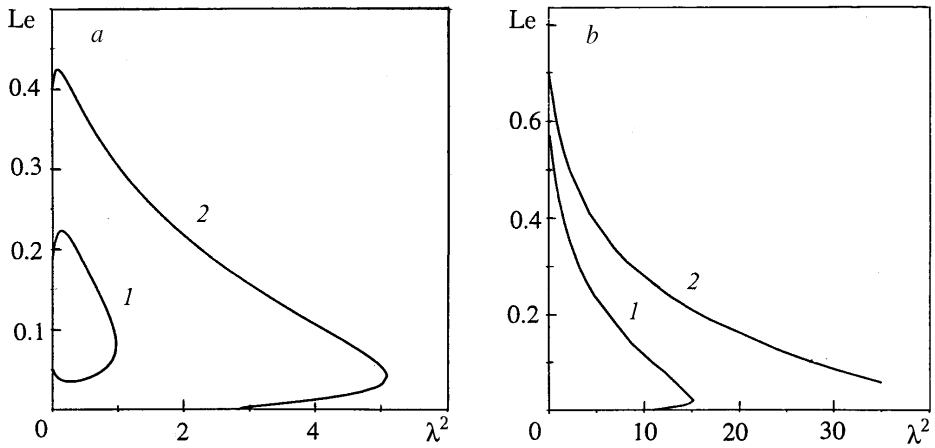

Fig. 2. Change in the region of periodic stability loss ($\mathrm{Le} < 1$) with increase in the dimensionless activation energy k: a) 5 (1), 7 (2); b) 10 (1), 15 (2). For $k = 5$, the instability island is formed by curve 1 and by a part of the Le axis, while for $k = 7$, 10, and 15, it is additionally formed by a part of the λ^2 axis.

Assuming in Eq. (27) that $\Omega \approx \lambda + \varepsilon/k$, where ε is the parameter to be determined, and using the expansions $\gamma \approx \lambda + \dfrac{\varepsilon}{k(2\lambda + 1)}$ and $\beta - \dfrac{1}{\mathrm{Le}} \approx \lambda + \dfrac{\varepsilon}{k(2\mathrm{Le}\,\lambda + 1)}$, for ε we obtain the expression

$$\varepsilon = -\frac{(2\lambda + 1)^2\,(2\,\mathrm{Le}\,\lambda + 1)}{2\,(\mathrm{Le} - 1)}\,.$$

In [5], it has been revealed that for $\mathrm{Le} > 1$ there are regions with vibrational and nonvibrational losses of stability. The boundary of the region for the first case is found from the condition $\Omega = 0$ and for the second one from the condition $\Omega = i\omega$, $i = \sqrt{-1}$. The parameter ω has the meaning of frequency. But if $\mathrm{Le} < 1$, only the vibrational stability loss is observed. However, the investigation of the stability carried out on the basis of formula (25) has shown a somewhat different picture. Thus, for $\mathrm{Le} > 1$ only the nonvibrational (aperiodic) stability loss is observed. The results of the numerical analysis of Eq. (25) are presented in Fig. 1. Here, the regions of unstable burning occupying the space above curves 1, 2, etc., for different values of the dimensionless activation energy k have the shape of peninsulas with a common shore, i.e., the ordinate Le. But if $\mathrm{Le} < 1$, the stability loss is only of a vibrational nature; the region of unstable burning has the shape of an island whose dimensions grow rapidly with increase in k. The results of the numerical calculation are given in Fig. 2.

Stability of Burning at a Variable Flame Velocity. It would seem that the indices $\Omega_1 = -\lambda^2$ and $\Omega_2 = -\lambda^2 - 1/4$ of the perturbation growth with negatively determined signs, found above for the case $\mathrm{Le} = 1$, can indicate the absolute stability of the flame with a variable v_{n}. Nonetheless, it turns out that apart from Ω_1 and Ω_2 we have one more regime of burning with the increment Ω_3 which is overlooked if we immediately set $\mathrm{Le} = 1$ in Eq. (21). Indeed, let in Eq. (21)

$$q = \frac{\Omega}{\lambda^2} + \mathrm{Le}\,. \tag{28}$$

In the equality

$$\gamma\frac{\Omega + \mathrm{Le}\,\lambda^2}{\Omega + \lambda^2} - \alpha - \gamma - (k - \gamma)\,\frac{k\,(k + 1) - (\Omega + \mathrm{Le}\,\lambda^2)}{k\,(k + 1) - (\Omega + \lambda^2)} + k + 1 = 0 \tag{29}$$

resulting from Eq. (21), we take $\Omega \equiv \Omega_3 = -\lambda^2 + k(k + 1)$. Evaluating the indeterminacy of the form 0/0 which arises in this case in the fraction $(k - \gamma)/[k(k + 1) - (\Omega_3 + \lambda^2)]$, we reduce Eq. (29) to the form

$$\lambda^2 \frac{k\,(\mathrm{Le}-1)}{(k+1)\,(2k+1)} = 0\ . \qquad (30)$$

The equality to zero of the left-hand side of Eq. (30) required here is possible for $\mathrm{Le} = 1$, if the exotic case $\lambda = 0$ corresponding to one-dimensional perturbations and the unreal case $k = \infty$ are neglected.

When the increment Ω_3 is present, the indeterminacy of the form 0/0 that is easily evaluated also arises in the solutions of F_1 and F_2. The unwieldy but simple calculation gives ($\mathrm{Le} = 1$)

$$x < 0: \ \ F_1 = G_1 = -\frac{k}{k+1} \exp x\ , \quad x > 0: \ \ F_2 = G_2 = -\frac{k}{k+1} \exp(-kx)\ .$$

Thus, for the wave numbers λ satisfying the inequality

$$\lambda < \sqrt{k\,(k+1)} \approx k = \frac{n-1}{\sqrt{n}} \frac{E}{RT_{\mathrm{b}}}$$

the flame is unstable.

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NOTATION

D and κ, coefficients of diffusion and thermal diffusivity; T_{b} and T_0, burning temperature and initial temperature of the mixture; E, activation energy of the chemical reaction; R, universal gas constant; N and N_0, running and initial concentrations of the reacting substance; k_0, pre-exponential factor in the Arrhenius law; η, Heaviside unit function; Q, thermal effect of the chemical reaction; c_p, heat capacity of the gas at constant pressure; x', coordinate of the direction of motion of the plane flame front. Subscript: b, burning.

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